RADIATIVE COOLING OF DROPLETS IN A POLYDISPERSE STREAM

É. A. Iodko and Ya. R. Élimelakh

UDC 536.3:532.529.5

On the basis of physical hypotheses, the authors propose a deterministic model of a stream of droplets resulting from the fragmentation of a jet. The radiative energy losses here are calculated on the basis of the geometrical–optic approximation.

The problem concerning the magnitude of thermal radiation losses in a stream of droplets arises in some metallurgical situations where the crystallization of ingots and castings is to be accelerated. An attempt was made in [1, 2] to solve this problem for the case of monodisperse streams.

In reality, however, streams of droplets are polydisperse. For this reason, the authors propose here a deterministic model of an axially symmetric stream resulting from the fragmentation of a jet and containing droplets of various sizes.

It is assumed that, where the jet fragmentation occurs, the vertical velocity component of all droplets is the same and equal to the jet velocity. At any other section along the stream the vertical velocity component is affected by the force of gravity only.

The radial momentum component is also assumed the same for droplets of any size, namely

$$\frac{4}{3}\pi\delta^3\rho U_r^{(\delta)} = \text{idem} \tag{1}$$

and to be determined by the mode of jet fragmentation. Finally, the trajectories of droplets inside the stream are assumed not to intersect.

It follows from all these assumptions that the size of droplets decreases monotonically along the radius of the stream. The smallest droplets are on the periphery and the largest droplets are in the central region. Inside the stream there is an axially symmetric cavity not containing any droplets. Based on the authors' observations, all streams of droplets resulting from the fragmentation of a jet by various methods have such a structure — at least qualitatively.

For the purpose of calculations according to our scheme, we must stipulate the following quantities:

1. the cross section area F_0 of the jet before fragmentation, or the radius R_0 of the "inlet section" of the stream;
2. the volume flow rate Q_0 of the metal;
3. the "aperture" angle 2α of the stream of droplets near the inlet section;
4. the temperature T_0 of droplets at the inlet section, the emissivity ε_0 of droplets, the emissivity ε_c of the "chiller," the surface temperature T_c of the chiller, and the surface area F_c of the chiller;
5. the length H of the droplet stream;
6. the size-distribution function of droplets at the inlet section
$$\delta = \delta(r');$$
7. the concentration distribution of droplets at the inlet section
$$N = N(r').$$

State University, Donetsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 24, No. 5, pp. 798-802, May, 1973. Original article submitted October 5, 1972.

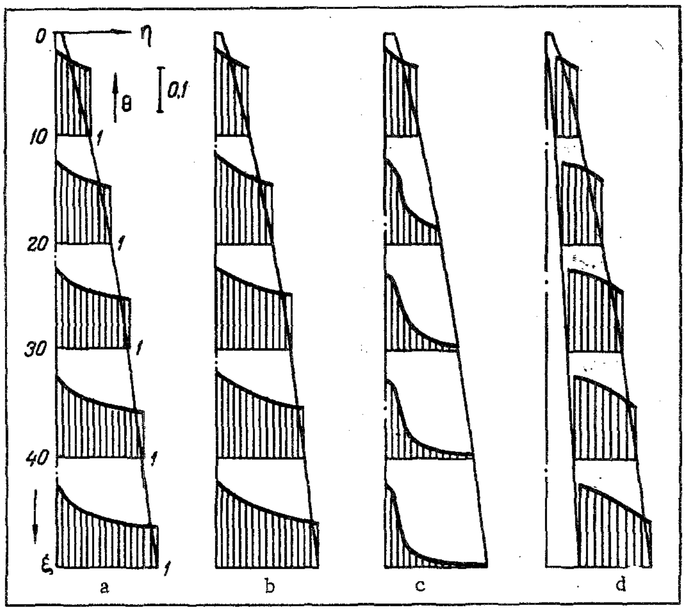

Fig. 1. Distribution of droplet temperatures along the height and across a section of the stream: (a) basic variant; (b) $N = \omega(r')^5$; (c) $\delta_{max} = 1$ mm and $\delta_{min} = 0.1$ mm; (d) $\delta_{max} = 0.6$ mm and $\delta_{min} = 0.4$ mm.

It follows from elementary considerations of balance that functions $\delta(r')$ and $N(r')$ are interrelated so as to limit the freedom in stipulating function $N(r')$ arbitrarily. This relation signifies the requirement that the flow rate of metal be the same, whether expressed in terms of the distribution function $\delta(r')$ and concentration $N(r')$ at the inlet section or expressed in terms of "global" parameters: the radius of the jet inlet section and the jet velocity at that section

$$2\pi U_z(0)\int_0^{R_0} \frac{4}{3}\pi\delta^3(r')N(r')r'dr' = \pi R_0^2 U_z(0).$$

Under the given assumptions, the trajectories of all droplets of any given fixed size form a completely defined axially symmetric surface. The equation of such a surface in dimensionless coordinates will be

$$\eta(l, \xi) = \eta_1(l) + \frac{2}{x^2}\operatorname{tg}\alpha\left(\frac{l_{\min}}{l}\right)^3[x\sqrt{\xi} - \ln(1 + x\sqrt{\xi})].$$

The equation of the outer surface of the stream, formed by the smallest droplets, is

$$\eta_{\max}(\xi) = 1 + \frac{2}{x^2}\operatorname{tg}\alpha\,[x\sqrt{\xi} - \ln(1 + x\sqrt{\xi})], \tag{3a}$$

and the equation of the surface of the inner cavity is

$$\eta_{\min}(\xi) = (\eta_{\max} - 1)\left(\frac{l_{\min}}{l_{\max}}\right)^3. \tag{3b}$$

The velocity of a droplet with the dimension l in any horizontal plane ξ is in dimensionless form:

$$\bar{V} = \bar{e}_r(l_{\min}/l)^3\operatorname{tg}\alpha + \bar{e}_z(1 + x\sqrt{\xi}). \tag{4}$$

The concentration of droplets in any horizontal plane is

$$c[\eta(l, \xi), \xi] = \frac{1}{1 + x\sqrt{\xi}}\,\frac{\eta_1(l)}{\eta(l, \xi)}\,c_1[\eta_1(l)]\,\frac{d\eta_1(l)}{dl}\Big/\frac{\partial\eta(l, \xi)}{\partial l}. \tag{5}$$

The surface formed by the trajectories of all droplets with the dimension l has an area

$$S(l) = 2\pi\int_0^h \eta(l, \xi)\sqrt{1 + \left(\frac{\partial\eta}{\partial\xi}\right)^2}\,d\xi. \tag{6}$$

The total area of the median section of the volume of droplets bounded by surface $S(l)$ and the outer surface of the stream is

$$F_\Sigma(l) = 2\pi^2 \int\limits_{l}^{l_{\min}} \left[\int\limits_0^h \sqrt{1 + \mathrm{tg}^2\alpha \left(\frac{l_{\min}}{l_1}\right)^6 \frac{1}{(1+x\sqrt{\xi})^2}}\, d\xi \right] \times l_1^2 \eta_1(l_1) \frac{d\eta_1(l_1)}{dl_1} c_1[\eta_1(l_1)]\, dl_1. \tag{7}$$

The mean-over-the-path referred emissivity of a droplet with the radius l has been calculated according to the "shielding" formula [3], under the assumption that all droplets surrounding a fixed "trajectory surface" may be regarded approximately as an array of "shields" with a total surface area F_Σ:

$$\varepsilon_{\mathrm{ref}}(l) = \varepsilon_0 \Bigg/ \left\{ \left[1 + \varepsilon_0 \left(\frac{1}{\varepsilon_c} - 1 \right) \frac{S(l)}{f_c} \right] \left[1 + \frac{F_\Sigma(l)}{S(l)} \right] \right\}. \tag{8}$$

The heat losses due to radiation from droplets of any size l at a distance ξ from the inlet section are

$$q(l, \xi) = \mathrm{Sk}\, l^2 \varepsilon_{\mathrm{ref}}(l) \int\limits_0^\xi [\theta^4(l, \xi) - \theta_x^4] \frac{1}{1 + x\sqrt{\xi}}\, d\xi. \tag{9}$$

Assuming now the largest size of droplets not to be excessive and the thermal conductivity of the droplet material (metal) to be high, one may disregard the temperature gradient inside a droplet. The dimensionless temperature of a droplet of size l in the horizontal plane ξ will then be determined according to the formula:

$$\theta(l, \xi) = \begin{cases} \dfrac{1}{\gamma} \left[\dfrac{i_0(l) - q(l, \xi)}{l^3} - K - 1 \right] + 1, & i_0 - q > l^3(K + 1); \\ 1; & l^3 \leqslant i_0 - q \leqslant l^3(K + 1); \\ \dfrac{i_0(l) - q(l, \xi)}{l^3}; & i_0 - q < l^3; \end{cases} \tag{10}$$

where

$$i_0(l) = l^3[\gamma(\theta_0 - 1) + K + 1]. \tag{11}$$

Equations (9) and (10) must be solved simultaneously. Numerical calculations were actually made on a "Dnepr" computer. As the basic "variant" we had selected the following one: the droplet material to be steel with ρ = 7000 kg/m^3, T_s = 1753°K, R_0 = 0.03 m, Q_0 = 0.008 m^3/sec (approximately 10 tons in 3 min), α = 45°, T_0 = 1823°K, ε_0 = 0.5, ε_c = 1.0, T_c = 273°K, F_c = 10^6 m^2, H = 1.5 m, $\delta_{\max}$ = 3 mm, $\delta_{\min}$ = 0.3 mm,

$$r' = \frac{\delta_{\max} - \delta}{\delta_{\max} - \delta_{\min}} R_0; \qquad N(r') = \omega r'.$$

Parameter ω was determined from condition (2).

The results of these computations are shown in Fig. 1a. We computed the concentration distribution of droplets at the inlet section according to the law $N(r') = \omega(r')^5$ (Fig. 1b), the decrease in droplet size to one third at a constant ratio $\delta_{\min}/\delta_{\max}$ (Fig. 1c), the decrease in ratio $\delta_{\min}/\delta_{\max}$ at a constant average droplet size (Fig. 1d), and the "aperture" angle of the jet was increased up to 60°.

The results show that decreasing the size of droplets will accelerate their cooling. Methods of jet fragmentation yielding a maximum quantity of droplets at the jet periphery (Fig. 1b) are somewhat preferable, inasmuch as the temperature of droplets will then be distributed more uniformly over a stream section. Still more uniform cooling of droplets in a stream is attained when this stream is made a nearly monodisperse one (Fig. 1d). Then, unlike in the other variants, a sizable cavity forms in the central region of the stream. Increasing the aperture angle also causes droplets to cool more uniformly over a stream section.

NOTATION

$\tilde{U}_0 = Q_0/\pi R_0^2$	is the characteristic velocity;
r, z	are the axial coordinates;

r'	is the radial coordinate at the "inlet section" of a stream;
δ	is the droplet radius;
U_z, U_r	are the vertical and the radial velocity components;
g	is the gravity acceleration;
σ	is the Stefan—Boltzmann constant;
$I_0^{(\delta)}$	is the heat content in a droplet of radius δ at temperature $T = T_0$;
ρ_L	is the density of the liquid material;
ρ_S	is the density of the solid material;
c_L	is the specific heat of the liquid material;
c_S	is the specific heat of the solid material;
ρ	is the specific heat of crystallization (solidification);
$\tilde{I}_0 = 4/3\pi R_0 \rho_S c_S T_S$	is the characteristic value of the heat content;
T_S	is the crystallization (solidification) temperature;

$\eta_1 = r/R_0$; $\eta = r'/R_0$; $\xi = z/R_0$; $l = \delta/R_0$; $V_\xi = U_z/\tilde{U}_0$; $V_\eta = U_r^{(\delta)}/\tilde{U}_0$; $x = \sqrt{2gR_0}/\tilde{U}_0$; $\gamma = \rho_L c_L/\rho_S c_S$; $K = \rho/c_S T_S$; $i_0^{(2)} \doteq I_0^{(\delta)}/I_0$; $Sk = (3\sigma T_S^3)/(2\tilde{U}_0 \rho_S c_S)$; $\theta = T/T_S$; $c_1(\eta_1) = (4/3)\pi N(r')R_0^3$; $f_c = F_c/R_0^2$.

Subscripts

L	refers to liquid;
S	refers to solid;
o	refers to "inlet section";
c	refers to "chiller";
ref	means referred.

LITERATURE CITED

1. É. A. Iodko, in: Steel Making, Trans. Donetsk Scientific-Research Inst. of Ferrous Metallurgy [in Russian], Izd. Metallurgiya (1966), Vol. 4, p. 268.
2. V. A. Zhuravlev and M. K. Trukhin, Inzh. Fiz. Zh., 21, No. 5 (1971).
3. A. G. Blokh, Fundamentals of Radiative Heat Transfer [in Russian], Gosénergoizdat, Moscow —Leningrad (1962).